

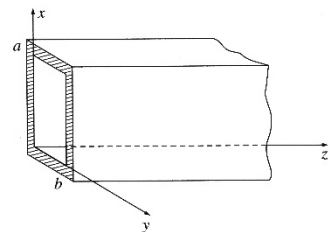
◇ Useful formulas: One A4 double-sided paper with handwriting.

1. (20%) Write the equations (if possible) and explain the following terms as clear as possible.

- (a) The Lorentz gauge and the Coulomb gauge. (4%)
- (b) Gauge transformations and gauge freedom. (4%)
- (c) Hidden momentum (4%)
- (d) The two postulates of the special relativity (4%)
- (e) Invariant quantity and conserved quantity. (4%)

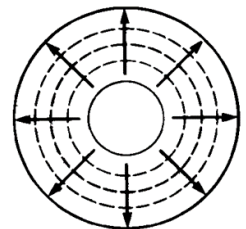
2. (20%) A wave is propagating in a rectangular waveguide with TE₁₀ mode. $B_z(x, z, t) = B_0 \cos(\pi x / a) \cos(kz - \omega t)$

- (a) Find E_x and B_y ? (10%) [Hint: Express in real components.] (10%)
- (b) Estimate the cutoff frequency in GHz of the TE₁₀ mode with width $a = 7.112$ mm and height $b = 3.556$ mm. [Hint: i.e., WR-28 waveguide] (10%)

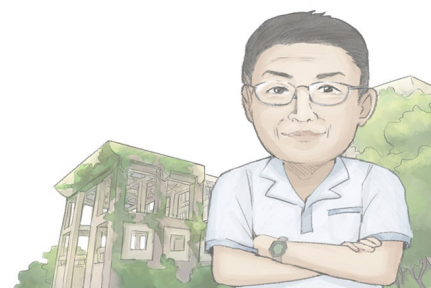


3. (20%) A coaxial cable usually works in the TEM mode with the fields

$$\mathbf{E}(s, \phi, z, t) = \frac{A \cos(kz - \omega t)}{s} \hat{\mathbf{s}} \quad \text{and} \quad \mathbf{B}(s, \phi, z, t) = \frac{A \cos(kz - \omega t)}{cs} \hat{\boldsymbol{\phi}}$$



- (a) Find the time averaged Poynting vector $\langle \mathbf{S} \rangle$ and the energy density $\langle u \rangle$. [Hint: Integrate over the cross section of the coaxial cable with the inner radius a and the outer radius b to get the energy per unit time and per unit length carried by the wave.] (10%)
- (b) Confirm that the energy in the coaxial cable travels at the group velocity. (10%)



4. (25%) A point charge q moves in a circle of radius a at constant angular velocity ω . Assume the circle lies in xy plane, centered at the origin, and at time $t=0$ the charge is at $(0, a)$, on the positive y axis.

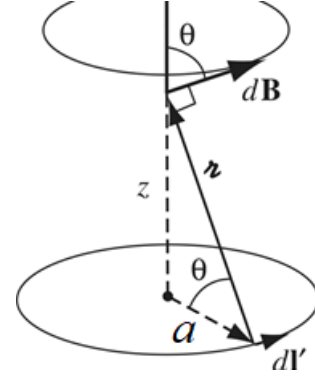
(a) Find the Lienard-Wiechert potentials for the points on the z axis. (8%)

(b) Calculate B_z on the z axis as functions of z and t . [Hint: $\mathbf{B} = \nabla \times \mathbf{A}$] (8%)

(c) Consider a circular loop of radius a , which carries a steady current I . The magnetic field at distance z above the center is

$$B(z) = \frac{\mu_0 I}{2} \frac{a^2}{(a^2 + z^2)^{3/2}}$$

Compare the results shown in (c) with those in (b). (9%)



5. (25%)

(a) Show that $(\mathbf{E} \cdot \mathbf{B})$ is relativistically invariant. (8%)

(b) Show that $(E^2 - c^2 B^2)$ is relativistically invariant. (8%)

(c) The relativistic transformation of EM fields between a rest frame K and a moving frame K' with velocity $\mathbf{v}$ and Lorentz factor γ are:

$$\begin{cases} \mathbf{E}'_{\parallel} = \mathbf{E}_{\parallel} \\ \mathbf{E}'_{\perp} = \gamma \left(\mathbf{E}_{\perp} + \frac{\mathbf{v}}{c} \times \mathbf{B}_{\perp} \right) \end{cases} \quad \text{and} \quad \begin{cases} \mathbf{B}'_{\parallel} = \mathbf{B}_{\parallel} \\ \mathbf{B}'_{\perp} = \gamma \left(\mathbf{B}_{\perp} - \frac{\mathbf{v}}{c} \times \mathbf{E}_{\perp} \right) \end{cases}$$

Find the velocity $\mathbf{v}_0$ such that the transverse electric components $\mathbf{E}'_{\perp}$ disappear. (9%)

